**C.S.DAV PUBLIC SCHOOL**

CLASS: 9th

Notes of Maths

Chapters: 01 & 02

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**Chapter:- 1**

**Introduction to Natural Numbers**

Non-negative counting numbers excluding zero are called **Natural Numbers**.

N = 1, 2, 3, 4, 5, ……….

**Whole Numbers**

All natural numbers including zero are called **Whole Numbers**.

W = 0, 1, 2, 3, 4, 5, …………….

**Integers**

All natural numbers, negative numbers and 0, together are called **Integers**.

Z = – 3, – 2, – 1, 0, 1, 2, 3, 4, …………..

**Rational Numbers**

The number ‘a’ is called **Rational** if it can be written in the form of r/s where ‘r’ and ‘s’ are integers and s ≠ 0,

Q = 2/3, 3/5, etc. all are rational numbers.

**How to find a rational number between two given numbers?**

To find the rational number between two given numbers ‘a’ and ‘b’.

rational

**Example:**

Find 2 rational numbers between 4 and 5.

**Solution:**

To find the rational number between 4 and 5

 rational number 

To find another number we will follow the same process again.

another number

Hence the two rational numbers between 4 and 5 are 9/2 and 17/4.

**Remark:**There could be unlimited rational numbers between any two rational numbers.

**Irrational Numbers**

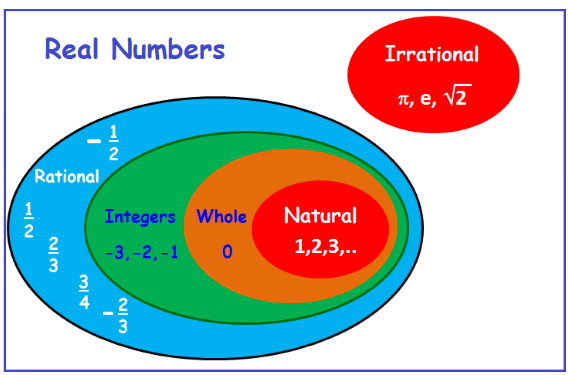
The number ‘a’ which cannot be written in the form of p/q is called irrational, where p and q are integers and q ≠ 0 or you can say that the numbers which are not rational are called **Irrational Numbers**.

**Example -** √7, √11 etc.

**Real Numbers**

All numbers including both rational and irrational numbers are called **Real Numbers**.

R = – 2, – (2/3), 0, 3 and √2



**Real Numbers and their Decimal Expansions**

**1. Rational Numbers**

If the rational number is in the form of a/b then by dividing a by b we can get two situations.

**a. If the remainder becomes zero**

While dividing if we get zero as the remainder after some steps then the decimal expansion of such number is called terminating.

**Example:**

7/8 = 0.875

**b. If the remainder does not become zero**

While dividing if the decimal expansion continues and not becomes zero then it is called non-terminating or repeating expansion.

**Example:**

1/3 = 0.3333….

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Hence, the decimal expansion of rational numbers could be terminating or non-terminating recurring and vice-versa.

**2. Irrational Numbers**

If we do the decimal expansion of an irrational number then it would be **non –terminating non-recurring**and vice-versa. i. e. the remainder does not become zero and also not repeated.

**Example:**

π = 3.141592653589793238……

**Representing Real Numbers on the Number Line**

To represent the real numbers on the number line we use the process of successive magnification in which we visualize the numbers through a magnifying glass on the number line.

**Example:**

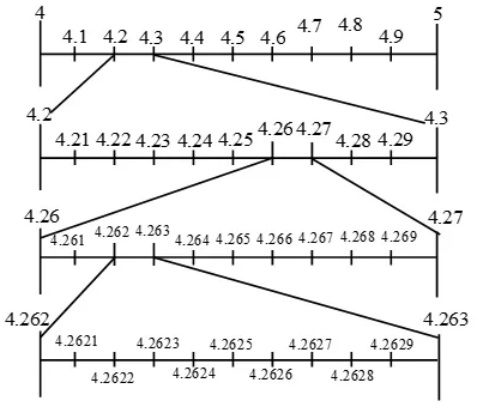
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**Step 1:** The number lies between 4 and 5, so we divide it into 10 equal parts. Now for the first decimal place, we will mark the number between 4.2 and 4.3.

**Step 2:** Now we will divide it into 10 equal parts again. The second decimal place will be between 4.26 and 4.27.

**Step 3:** Now we will again divide it into 10 equal parts. The third decimal place will be between 4.262 and 4.263.

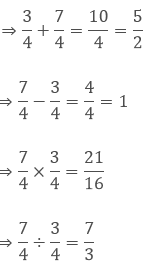
**Step 4:** By doing the same process again we will mark the point at 4.2626.



**Operations on Real Numbers**

1. The sum, difference, product and quotient of two rational numbers will be rational.

**Example:**



2. If we add or subtract a rational number with an irrational number then the outcome will be irrational.

**Example:**

If 5 is a rational number and √7 is an irrational number then 5 + √7 and 5 - √7 are irrational numbers.

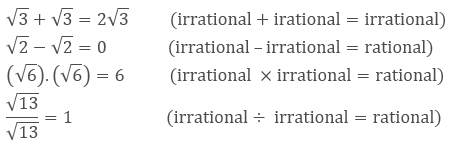
3. If we multiply or divide a non-zero rational number with an irrational number then also the outcome will be irrational.

**Example:**

If 7 is a rational number and √5 is an irrational number then 7√7 and 7/√5 are irrational numbers.

4. The sum, difference, product and quotient of two irrational numbers could be rational or irrational.

**Example:**



**Finding Roots of a Positive Real Number ‘x’ geometrically and mark it on the Number Line**

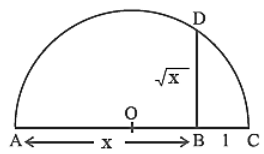
To find √x geometrically

1. First of all, mark the distance x unit from point A on the line so that AB = x unit.

2. From B mark a point C with the distance of 1 unit, so that BC = 1 unit.

3. Take the midpoint of AC and mark it as O. Then take OC as the radius and draw a semicircle.

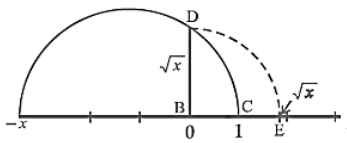
4. From the point B draw a perpendicular BD which intersects the semicircle at point D.



**The length of BD** **= √x.**

To mark the position of √x on the number line, we will take AC as the number line, with B as zero. So C is point 1 on the number line.

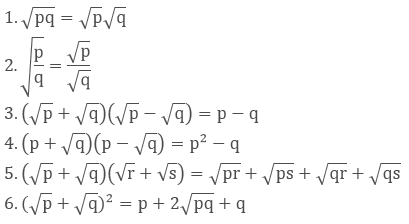
Now we will take B as the centre and BD as the radius, and draw the arc on the number line at point E.



Now E is √x on the number line.

**Identities Related to Square Roots**

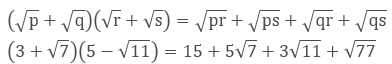
If p and q are two positive real numbers



**Examples:**

1. SimplifySimplify

We will use the identity



2. Simplify Simplify

We will use the identity

use

**Rationalizing the Denominator**

Rationalize the denominator means to convert the denominator containing square root term into a rational number by finding the equivalent fraction of the given fraction.

For which we can use the identities of the real numbers.

**Example:**

Rationalize the denominator of 7/(7- √3).

**Solution:**

We will use the identityequationhere.

numbers

**Laws of Exponents for Real Numbers**

If we have a and b as the base and m and n as the exponents, then

1. am × an =am+n

2. (am)n = amn

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4. ambm = (ab)m

5.a0 = 1

6. a1 = a

7. 1/an = a-n

* Let a > 0 be a real number and n a positive integer.

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* Let a > 0 be a real number. Let m and n be integers such that m and n have no common factors other than 1, and n > 0. Then,

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**Example:**

Simplify the expression (2x3y4) (3xy5)2.

**Solution:**

Here we will use the law of exponents

am × an =am+n and (am)n = amn

(2x3y4)(3xy5)2

(2x3y4)(3 2 x 2 y10)

18. x3. x2. y4. y10

18. x3+2. y4+10

18x5y14

**CHAPTER:- 2**

**Polynomial**

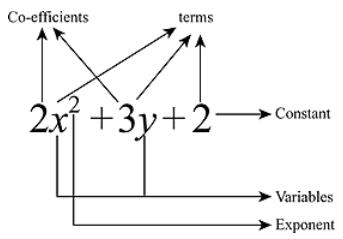
Polynomial is an algebraic expression which includes constants, variables and exponents. It is the expression in which the variables have only positive integral powers.

**Example**

1. 4x3 + 3x2 + x +3 is a polynomial in variable x.

2. 4x2 + 3x-1 - 4 is not a polynomial as it has negative power.

3. 3x3/2+ 2x – 3 is not a polynomial.



* Polynomials are denoted by p(x), q(x) etc.
* In the above polynomial 2x2, 3y and 2 are the terms of the polynomial.
* 2 and 3 are the coefficient of the x2 and y respectively.
* x and y are the variables.
* 2 is the constant term which has no variable.

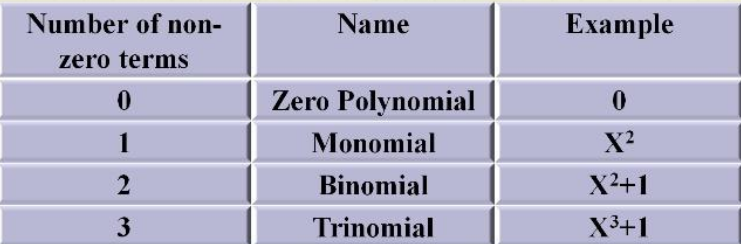
**Polynomials in One Variable**

If there is only one variable in the expression then this is called the polynomial in one variable.

**Example**

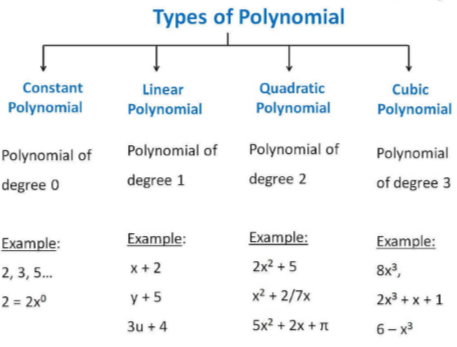
* x3 + x – 4 is polynomial in variable x and is denoted by p(x).
* r2 + 2 is polynomial in variable r and is denoted by p(r).

**Types of polynomials on the basis of the number of terms**



**Types of polynomials on the basis of the number of degrees**

The highest value of the power of the variable in the polynomial is the degree of the polynomial.



**Zeroes of a Polynomial**

If p(x) is a polynomial then the number ‘a’ will be the zero of the polynomial with p(a) = 0. We can find the zero of the polynomial by **equating it to zero**.

**Example: 1**

Given polynomial is p(x) = x - 4

To find the zero of the polynomial we will equate it to zero.

x - 4 = 0

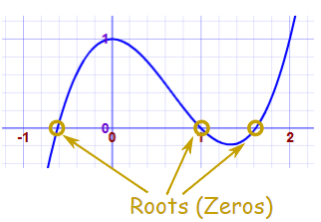
x = 4

p(4) = x – 4 = 4 – 4 = 0

This shows that if we place 4 in place of x, we got the value of the polynomial as zero. So 4 is the zero of this polynomial. And also we are getting the value 4 by equating the polynomial by 0.

So 4 is the zero of the polynomial or root of the polynomial.

The **root of the polynomial** is basically the **x-intercept** of the polynomial.



If the polynomial has one root, it will intersect the x-axis at one point only and if it has two roots then it will intersect at two points and so on.

**Example: 2**

Find p (1) for the polynomial p (t) = t2 – t + 1

p (1) = (1)2 – 1 + 1

= 1 – 1 + 1

= 1

**Remainder Theorem**

We know the property of division which follows in the basic division, i.e.

**Dividend = (Divisor × Quotient) + Remainder**

This same follows the division of polynomial.

If p(x) and g(x) are two polynomials in which the degree of p(x) ≥ degree of g(x) and g(x) ≠ 0 are given then we can get the q(x) and r(x) so that:

**P(x) = g(x) q(x) + r(x),**

where r(x) = 0 or degree of r(x) < degree of g(x).

It says that p(x) divided by g(x), gives q(x) as quotient and r(x) as remainder.

Let’s understand it with an example

**Division of a Polynomial with a Monomial**

Division of a Polynomial with a Monomial

We can see that ‘x’ is common in the above polynomial, so we can write it as

common

Hence 3x2+ x + 1 and x the factors of 3x3 + x2 + x.

**Steps of the Division of a Polynomial with a Non –Zero Polynomial**

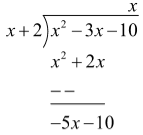
Divide x2 - 3x -10 by 2 + x

**Step 1:**  Write the dividend and divisor in the descending order i.e. in the standard form. x2 - 3x -10 and x + 2

Divide the first term of the dividend with the first term of the divisor.

x2/x = x this will be the first term of the quotient.

**Step 2:** Now multiply the divisor with this term of the quotient and subtract it from the dividend.



**Step 3:** Now the remainder is our new dividend so we will repeat the process again by dividing the dividend with the divisor.

**Step 4:**– (5x/x) = – 5

**Step 5:**



The remainder is zero.

Hence x2 - 3x – 10 = (x + 2)(x - 5) + 0

Dividend = (Divisor × Quotient) + Remainder

**Remainder Theorem** says that if p(x) is any polynomial of degree greater than or equal to one and let ‘t’ be any real number and p (x) is divided by the linear polynomial x – t, then the remainder is p(t).

As we know that

P(x) = g(x) q(x) + r(x)

If p(x) is divided by (x-t) then

If x = t

P (t) = (t - t).q (t) + r = 0

To find the remainder or to check the multiple of the polynomial we can use the remainder theorem.

**Example:**

What is the remainder if a4 + a3 – 2a2 + a + 1 is divided by a – 1.

**Solution:**

P(x) = a4 + a3 – 2a2 + a + 1

To find the zero of the (a – 1) we need to equate it to zero.

a -1 = 0

a = 1

p (1) = (1)4 + (1)3 – 2(1)2 + (1) + 1

= 1 + 1 – 2 + 1 + 1

= 2

So by using the remainder theorem, we can easily find the remainder after the division of polynomial.

**Factor Theorem**

Factor theorem says that if p(y) is a polynomial with degree n≥1 and t is a real number, then

1. (y - t) is a factor of p(y), if p(t) = 0, and
2. P (t) = 0 if (y – t) is a factor of p(y).

**Example: 1**

Check whether g(x) = x – 3 is the factor of p(x) = x3- 4x2+ x + 6 using factor theorem.

**Solution:**

According to the factor theorem if x - 3 is the factor of p(x) then p(3) = 0, as the root of x – 3 is 3.

P (3) = (3)3- 4(3)2 + (3) + 6

= 27 – 36 + 3 + 6 = 0

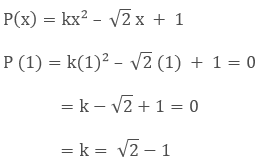
Hence, g(x) is the factor of p(x).

**Example: 2**

Find the value of k, if x – 1 is a factor of p(x) = kx2– √2x + 1

**Solution:**

As x -1 is the factor so p(1) = 0



**Factorization of Polynomials**

Factorization can be done by three methods

**1. By taking out the common factor**

If we have to factorize x2 –x then we can do it by taking x common.

x(x – 1) so that x and x-1 are the factors of x2 – x.

**2. By grouping**

ab + bc + ax + cx = (ab + bc) + (ax + cx)

= b(a + c) + x(a + c)

= (a + c)(b + x)

**3. By splitting the middle term**

x2 + bx + c = x2 + (p + q) + pq

= (x + p)(x + q)

This shows that we have to split the middle term in such a way that the sum of the two terms is equal to ‘b’ and the product is equal to ‘c’.

**Example: 1**

Factorize 6x2 + 17x + 5 by splitting the middle term.

**Solution:**

If we can find two numbers p and q such that p + q = 17 and pq = 6 × 5 = 30, then we can get the factors.

Some of the factors of 30 are 1 and 30, 2 and 15, 3 and 10, 5 and 6, out of which 2 and 15 is the pair which gives p + q = 17.

6x2 + 17x + 5 =6 x2 + (2 + 15) x + 5

= 6 x2 + 2x + 15x + 5

= 2 x (3x + 1) + 5(3x + 1)

= (3x + 1) (2x + 5)

|  |
| --- |
| **Algebraic Identities** |
| 1. (x + y)2= x2+ 2xy + y2 |
| 2. (x - y)2 = x2- 2xy + y2 |
| 3. (x + y) (x - y) = x2 - y2 |
| 4. (x + a) (x + b) = x2 + (a + b)x + ab |
| 5. (x + y + z)2= x2+ y2+ z2+ 2xy + 2yz + 2zx |
| 6. (x + y)3 = x3 + y3 + 3xy(x + y) = x3+ y3 + 3x2y + 3xy2 |
| 7. (x - y)3 = x3- y3 - 3xy(x - y) = x3- y3 - 3x2y + 3xy2 |
| 8. x3+ y3 = (x + y)(x2 – xy + y2) |
| 9. x3 - y3= (x - y)(x2 + xy + y2) |
| 10. x3+ y3 + z3- 3xyz = (x + y + z)(x2 + y2+ z2 – xy – yz - zx)       x3 + y3 + z3 = 3xyz if x + y + z = 0 |

**Example: 2**

Factorize 8x3 + 27y3 + 36x2y + 54xy2

**Solution:**

The given expression can be written as

= (2x)3+ (3y)3 + 3(4x2) (3y) + 3(2x) (9y2)

= (2x)3 + (3y)3 + 3(2x)2(3y) + 3(2x)(3y)2

= (2x + 3y)3 (Using Identity VI)

= (2x + 3y) (2x + 3y) (2x + 3y) are the factors.

**Example: 3**

Factorize 4x2 + y2 + z2 – 4xy – 2yz + 4xz.

**Solution:**

4x2 + y2 + z2 – 4xy – 2yz + 4xz = (2x)2+ (–y)2 + (z)2 + 2(2x) (-y)+ 2(–y)(z) + 2(2x)(z)

= [2x + (- y) + z]2 (Using Identity V)

= (2x – y + z)2 = (2x – y + z) (2x – y + z)

**……….COMPLETED……….**